10 marks **Computational Complexity Theory (B)** **1h20m**

Hardness is given as [x]. Higher number means harder. Marks as (x). Assume the input alphabet is $\{0, 1\}$ unless mentioned otherwise.

1. Define NL and show using your definition that NL $\subseteq$ P. [1] (1)
2. Show that if $A \leq^m_{\log} B$ and $B \leq^m_{\log} C$, then $A \leq^m_{\log} C$. [1] (1)
3. Define IP and show using your definition that Graph non-isomorphism is in IP. [1] (1)
4. Define a family of pair-wise independent hash functions and prove that the family is pair-wise independent. [2] (2)
5. We saw that NL is contained in the class of languages decided using $\log^2(n)$-space. Does this directly show NL $\subseteq$ P? Why or why not? [1] (1)
6. Show that directed graph reachability is $\leq^m_{\log}$-complete for NL. [1] (1)
7. Suppose in the given directed graph, each vertex is guaranteed to have out-degree at most one. Show that deciding graph reachability in such graphs is in LOGSPACE. [1] (1)
8. Describe a $\leq^m_{\log}$ reduction from directed graph reachability to directed graph reachability in a graph where every vertex has out-degree at most two. [2] (2)

Student's Scores

(To be filled by the instructor)

1	2	3	5	6	7	[1]
/1	/1	/1	/1	/1	/1	/6

4	8	[2]
/2	/2	/4

Instructor's Remarks

(To be filled by the instructor)