CCT Worksheet 1 **Feb 2026**

Basics

1. Show that for a language A, if there is a TM deciding it, then we can assume without loss of generality that there is a TM with exactly one accepting state deciding A.
2. Write an expression to upper bound the number of distinct configurations for a one-tape TM that uses at most k tape cells.
3. What is the domain and range for δ for a two-tape TM?
4. Same as above but for an NTM.
5. Are there languages A that can be decided in $t(n) = 0$ time? If so, are there languages decidable in time $t(n) = 1$ that are not decidable in $t(n) = 0$?
6. Let n be any natural number. Argue that for any language A, there is a TM that correctly decides $x \in A$ in time $|x|$ if $|x| \leq n$.
7. Suppose $A \in \mathrm{P}$ and a language B such that a string x is in B if and only if there is a string $y \in A$ and y differs from x at atmost one position (y has the same length as x). Show that $B \in \mathrm{P}$. Also if $A \in \mathrm{NP}$, show that $B \in \mathrm{NP}$.
8. Define an appropriate language for $n^2 \times n^2$ sudoku puzzles and show that it is in NP.

Intermediate

1. Argue that TMs with a one-sided infinite tape (the tape extends infinitely only to the right) is as powerful as ones with two-sided infinite tapes.
2. Suppose a language A is decided by a TM M that runs in $n^2 + k$ steps for $n \geq n_0$ for some constants n_0 and k. Show that A also has a TM that decides it in time $n^2/2 + k'$ steps for $n \geq n_0'$ for some constants n_0' and k'.
3. Consider the class of languages A such that there is a constant k and TM M that accepts all $x \in A$ in $n^k + k$ steps but on $x \notin A$ the TM M could reject or loop infinitely. Show that this class is the same as P,
4. Show that $\mathrm{NP} \subseteq \mathrm{coNP}$ implies $\mathrm{NP} = \mathrm{coNP}$.
5. Consider the language $\{x \mid x \text{ is not a palindrome}\}$. Show that this language can be decided by a two-tape NTM that runs in n time. The TM that we designed took $2n$ time. Provide your NTM the stay option.
6. Suppose you are given a Boolean formula F and a satisfying assignment x for F and you have to decide whether there is another satisfying assignment $y \neq x$. Show that this problem is $\leq_p^m$-complete for NP.
7. The same as above but for the vertex cover problem. You are given one vertex cover and have to decide whether or not there is a different vertex cover that is the same size.
8. Show that any language decided by an NTM that uses poly-space is also decided by a DTM that uses only poly-space.