10 marks **CCT Take-Home 1** **Feb 2026**

1. Define a TM that given the binary representation of a number outputs the square of that number. Test your TM on any TM simulator available online.
2. Show that CIRCUITSAT $\leq_p^m$ FORMULASAT.
3. Show that FORMULASAT $\leq_p^m$ 3SAT.
4. Show that 2SAT $\in$ P.
5. Let PRIMES $= \{n \mid n$ is the binary representation of a prime number$\}$. Show that this language is in NP. Does testing all numbers from 2 to $n-1$ for non-trivial divisors show that this is in P? Use the fact that if there is an $1 < a < n$ such that $a^{n-1} = 1 \pmod{n}$ and $a^{(n-1)/p} \neq 1 \pmod{n}$ for all primes p that divide $n-1$, then n is prime. Otherwise, n is 1 or 2 or composite.
6. Show that given a function that determines whether or not a given graph has a 3-colouring in poly-time, we can write a function that runs in poly-time and produces a 3-colouring for a given graph or output that none exists.
7. Given a string of parenthesis '(' and ')' and square brackets '[' and ']', check whether the string is properly balanced in log-space. Note that ([)] is not balanced.
8. A language A is downward self-reducible if $x \in A$ can be decided by a function for A that only decides $y \in A$ for strings y smaller than x. Show that all downward self-reducible languages are in PSPACE.
9. Design a log-space algorithm to check whether a given undirected graph has a cycle.
10. Design a log-space, many-one reduction from non-bipartiteness to undirected graph reachability. Non-bipartiteness is $\{G \mid G$ is not bipartite$\}$ and undirected graph reachability is $\{G \mid$ there is a path from 1 to n in $G\}$.